RAN-2101001102020001

M.A. (Part-II) Examination March - 2026

Mathematics - Numerical Analysis (Paper-502)

Time: 3 Hours] **[Total Marks: 100**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Answer all questions (3) Figures to the right indicates marks (4) Follow usual notations and conventions	Student's Signature

Q.1. Attempt all

1. If $\phi(x)$ is a continuous function in some interval $[a, b]$ that contains the root and $|\phi'(x)| \leq c < 1$ in this interval, then prove that for any choice of $x_0 \in [a, b]$, the sequence $\{x_k\}$ determined from $x_{k+1} = \phi(x_k)$, $k = 0, 1, 2, \ldots$ converges to the root ξ of $x = \phi(x)$. **07**

2. The equation $f(x) = x^4 - x - 10 = 0$ has a root in the internal (l, 2). Taking the initial approximation $x_0 = 1.5$, perform three iterations of the Chebyshev iteration method. **07**

3. Derive Gauss-Seidel Iteration Method for solving the system of equations. Give its Matrix form and Error form. **06**

OR

Q.1. Attempt all

1. Solve the system of equations : **07**

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = A$$

by the LU decomposition method with $u_{11} = u_{22} = u_{33} = l$.

2. For the following system of equations $\begin{bmatrix} -3 & 1 & 0 \\ 2 & -3 & 1 \\ 0 & 2 & -3 \end{bmatrix} X = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$ **07**

   (a) Set up the Gauss-Seidel iteration scheme in matrix form

   (b) Starting with $x^{(0)} = 0$, iterate two times.

3. Discuss Jacobi method to finding eigenvalues and eigenvectors for symmetric matrices. **06**

Q.2. **Attempt all**

1. Using the Jacobi method find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$ **07**
2. Discuss numerical differentiation method based on interpolation for uniform nodal points for linear and quadratic interpolation. **07**
3. For the method $f'(x_0) = \dfrac{-3f(x_0)+4f(x_1)-f(x_2)}{2h} + \dfrac{h^2}{3} f'''(\xi),\ x_0 < \xi < x_2$ **06**

   determine the optimal value of h, using the criteria
   (i) $|RE| = |TE|$
   (ii) $|RE| + |TE| = \text{minimum}$

OR

Q.2. **Attempt all**

1. If the general integral is of the type $\int_a^b w(x)\, f(x)\, dx$, then derive Newton-Cotes integration method (for $n = 1,2$) formula to find an approximate value of this integral. **07**
2. Determine a,b,c such that the formula $\int_0^h f(x)\, dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$ is exact for polynomials of as high order as possible, and determine the order of the truncation error. **07**
3. Evaluate the integral $I = \int_1^2 \dfrac{2x}{1+x^4}\, dx$ using Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. **06**

Q.3. **Attempt all**

1. Derive Simpson's rule for integration using method based on Undetermined Coefficients. **07**
2. Use the classical Runge-Kutta method of forth order to find the numerical solution at $x = 0.8$ for $\dfrac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$. Assume the step length $h = 0.2$. **07**
3. Solve the boundary value problem $u'' = 4u + 3x$, $0 < x < 1$, $u(0) = 1$, $u(1) = 1$ with $h = 0.25$, using the second order method. **06**

OR

Q.3. **Attempt all**

1. Discuss method based on interpolation for non-uniform nodal points for quadratic interpolation. **07**
2. Using the following data, find $f'(6.0)$ and $f''(6.3)$ by linear and quadratic interpolation. **07**

x	6.0	6.1	6.2	6.3	6.4
$f(x)$	0.1750	– 0.1998	– 0.2223	– 0.2422	– 0.2596

3. If $f^{(r)}(x)$ is expressed in the form $h^r f^{(r)}(x_k) = \sum_{-\rho}^{\rho} a_v f_{k+v}$ and points are equispaced with step length h, then derive method based on undetermined coefficients technique when $r = 2$ and $\rho = 2$. **06**

Q.4. Attempt all

1. Consider the four point formula $f'(x_2) = \frac{1}{6h}[-2f(x_1) - 3f(x_2) + 6f(x_3) - f(x_4)] + TE + RE$, where $x_j = x_0 + jh, j = 1, 2, 3, 4$ and TE, RE are respectively the truncation and round-off error. **07**
   a. Determine the form of TE and RE
   b. Obtain the optimum step length h satisying the criterion $|TE| = |RE|$
   c. Determine the total error

2. Let function $f(x, y)$ of two variables only. The values of the function $f(x, y)$ be given at a set of points (x_i, y_j) in the XY plane with spacing h and k in x and y directions respectively. We have $x_i = x_0 + ih$ and $y_j = y_0 + jk$; $i, j = 1,2,3,...,$then define $\left(\frac{\partial f}{\partial x}\right), \left(\frac{\partial f}{\partial y}\right), \left(\frac{\partial^2 f}{\partial x^2}\right), \left(\frac{\partial^2 f}{\partial y^2}\right)$. Hence prove that $\left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(xiyj)} = \left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\right)_{(xiyj)} = \left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right)_{(xiyj)}$ **07**

3. Given that **06**

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$y = f(x)$	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ and $x = 1.6$ by using finite difference method.

OR

Q.4. Attempt all

1. If the general integral is of the form $\int_a^b w(x) f(x)\, dx$, then derive Newton-Cotes integration methods formula to find an approximate value of this integral. **07**
2. Find the approximate value of $I = \int_{\pi/4}^{3\pi/2} \left(\frac{1}{sinx}\right)^{1/4} dx$ using (i) mid-point, (ii) three-point, and (iii) three-point open-type rules. **07**
3. Define Gauss quadrature integral and derive Gauss-Chebyshev two-point formula. **06**

Q.5. Attempt all

1. Calculate $\int_0^{1/2} \frac{x}{sinx} dx$ using Trapezoidal rule with $h = 1/2, 1/4, 1/8$ and Romberg integration. Assume $f(0)$ is taken as the limiting value. **07**
2. Find the three term Taylor series solution for the third order initial value problem $W''' + WW'' = 0, W(0) = 0, W'(0) = 0, W''(0) = 1$ **07**
   Find the bound on the error for $t \in [0,0.2]$.
3. Given the initial value problem $u' = -2tu^2$, $u(0) = 1$. Estimate $u(0.4)$ with $h = 0.2$ using classical fourth order Runge-Kutta method **06**

OR

Q.5. Attempt all

1. State general form of implicit multistep methods and hence derive Milne-Simpson methods for $k = 0,1,2$ **07**
2. A boundary value problem is defined as $y'' - y = 0$, where $y(0) = 0$, $y(2) = 3.62686$. With $h = 0.5$, use the finite difference method to solve. **07**
3. Solve boundary value problem $u'' = xu$, $u(0) + u'(0) = 1$, $u(1) = 1$ with $h = 1/3$ using Numerov method. **06**